Time-Dependent Stratified Flow Over Topography: Waves and Rotating Hydraulics

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LONG-TERM GOALS

My long-term goals for this research project are to understand the interaction of stratified flow with topography with an emphasis on those flows relevant to coastal oceans and marginal seas.

OBJECTIVES

The specific objectives of the last year have been a study of two related problems in the general area of time-dependent rotating hydraulics and nonlinear waves. The first is a study of the fully nonlinear dam break problem in a rotating channel. One focus of this work has been to study of the characteristics and dynamics of the shocks and bores. The second project is an extension of Long's (1954, 1970) classic problem of upstream influence and establishment of hydraulically control to flow in rotating channels. Both of these projects have been on single-layer flows, though the longterm goal is to extend the work to two-layer systems.

APPROACH

I have been carrying out this work using theoretical analysis and numerical modeling. Some laboratory experiments are planned.

WORK COMPLETED

Along with my coauthors L. Pratt (WHOI) and A. Kuo (Columbia), I have submitted a paper on nonlinear Rossby adjustment in a channel to *J. Fluid Mech.* (Helfrich, Pratt and Kuo, submitted). A second paper on the extension of Long's problem to rotating flow in a channel is in final stages of preparation and will soon be submitted to *J. Phys. Oceano*. My coauthors are L. Pratt and E. Chassignet (FSU).

RESULTS

Rossby adjustment in a channel:

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The classic dam break problem in a uniform width channel has been extended to include the effects of rotation. When the depth of the fluid is initially uniform upstream of the dam and zero downstream of it, the evolution of the flow can be obtained analytically by assuming geostrophic along-channel flow (semigeostrophy) using the characteristic method and simple wave theory. The nature of the flow has been investigated as a function of the sole parameter in the problem, the width of the channel relative to a deformation radius based on the initial upstream fluid depth. The fluid intrudes as a narrow current along the right-hand wall with a constant nose speed, in excess of the nonrotating limit (Stoker, 1957), that increases with channel width. Along the left-hand wall the flow separates from the boundary at a point that moves slowly downstream at a constant speed, which decreases as the width increases. Steady state solutions, including the mass transport, are found. Formal validity of the semigeostrophic approximation is expected for narrow channels; numerical computations of the full shallow water equations bear this idea out. Differences between the theory and computations increase as the channel width is increased.

When the downstream depth is finite the destruction of the barrier causes the upper fluid to intrude into the shallower fluid and the formation of a leading shock, or bore. Stoker (1957) gives the hydrostatic solution with no rotation. Gill (1976) examined the linear limit when rotation is included. A steady current is set up by Kelvin waves moving away from the barrier position. The Kelvin wave moving into the shallower layer is trapped to the right-hand wall (looking in the direction of the shallow layer) while the Kelvin wave moving into the deeper layer is trapped to the left wall. The steady flow approaches the section of the initial barrier along the left wall, crosses the channel at that section, and continues along the right wall.

When the initial depth difference is finite, the Kelvin wave moving into the shallow layer is replaced by a Kelvin bore. Because of the uncertainty of the proper shock-joining conditions and nonconservation of potential vorticity across discontinuities we employed numerical methods to study the flow. Figure 1a shows the depth field for a channel of nondimensional width $w = 4$ and an initial nondimensional depth in the shallow section $d_0 = 0.1$, which is scaled by the initial depth in the deep region D. The horizontal dimensions are normalized by the deformation radius based on D. The flow is shown at $t = 20$. In this example the leading shock curves back upstream and its amplitude decays away from the right wall. Trailing the bore is a geostrophic boundary current. Figure 1b shows a case in a narrow channel with $w = 1$ and $d_0 = 0.25$. The leading bore is now nearly straight across the channel. The amplitude decays away from the right wall ($y = -0.5$), but increases near the left wall. Trailing the bore near the left wall is a packet of Poincare waves.

We find that the leading bore extends across the channel to attach at the left wall (within one deformation radius of its position on the right wall) when the channel width scaled by the deformation radius in the upstream region, $wd_0^{-1/2} \leq 3$. In all cases the shock is connected to the following geostrophic current by an ageostrophic boundary layer characterized by a strong transverse jet (c.f., Fedorov and Melville, 1996). Shock amplitudes, measured on the right wall, and shock speeds increase above the nonrotating solutions as w is increased for a given d_0. However, the relation between bore speed and amplitude branches below the nonrotating relation as w (rotation) is increased. Finally, the potential vorticity jump across the shock is shown to depart significantly from the pseudo-inviscid

estimates due to the viscous flux of vorticity, an effect that is at best crudely parameterized and poorly understood.

Figure 1a. Layer depth for $w = 4$ and $d_J = 0.1$ at $t = 20$.

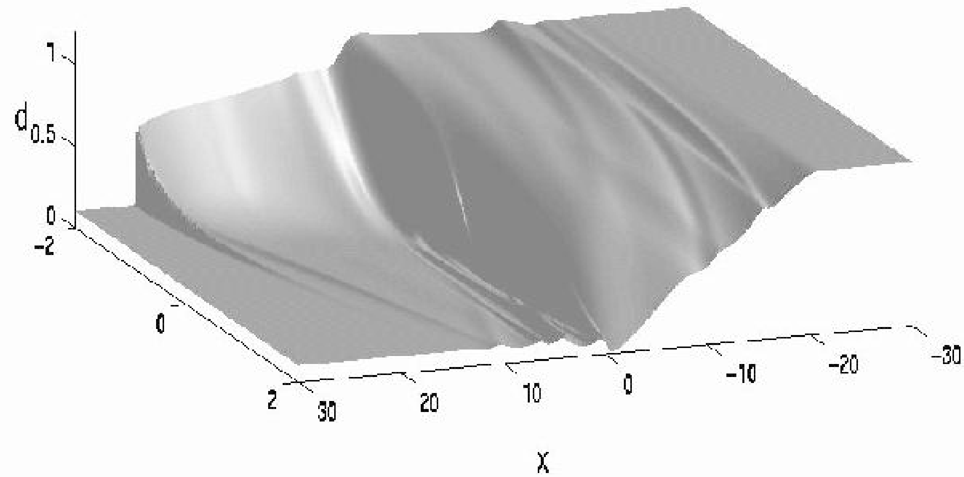
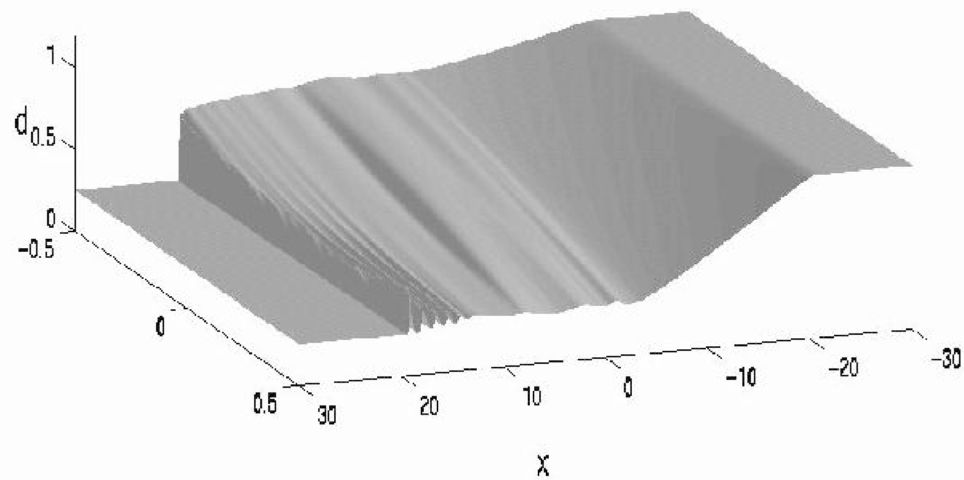


Figure 1b. Layer depth for $w = 1$ and $d_- = 0.25$ at $t = 20$.



Long's problem with rotation:

In Long's (1954, 1970) classic problem on upstream influence a topographic feature is rapidly grown into a nonrotating, hydrostatic, single layer flow. The response of the fluid, and in particular the development of upstream influence and subsequent establishment of hydraulic control at the sill crest, can be determined by the location of the flow in the F_d - h_m plane, where F_d is the Froude number of the initial flow and h_m is the final height of the topography. Upstream influence occurs through the propagation of a shock, or bore, which causes a permanent change to subcritical flow approaching the obstacle. At the crest of the obstacle the flow is critical and downstream a hydraulic jump back to subcritical flow may exist.

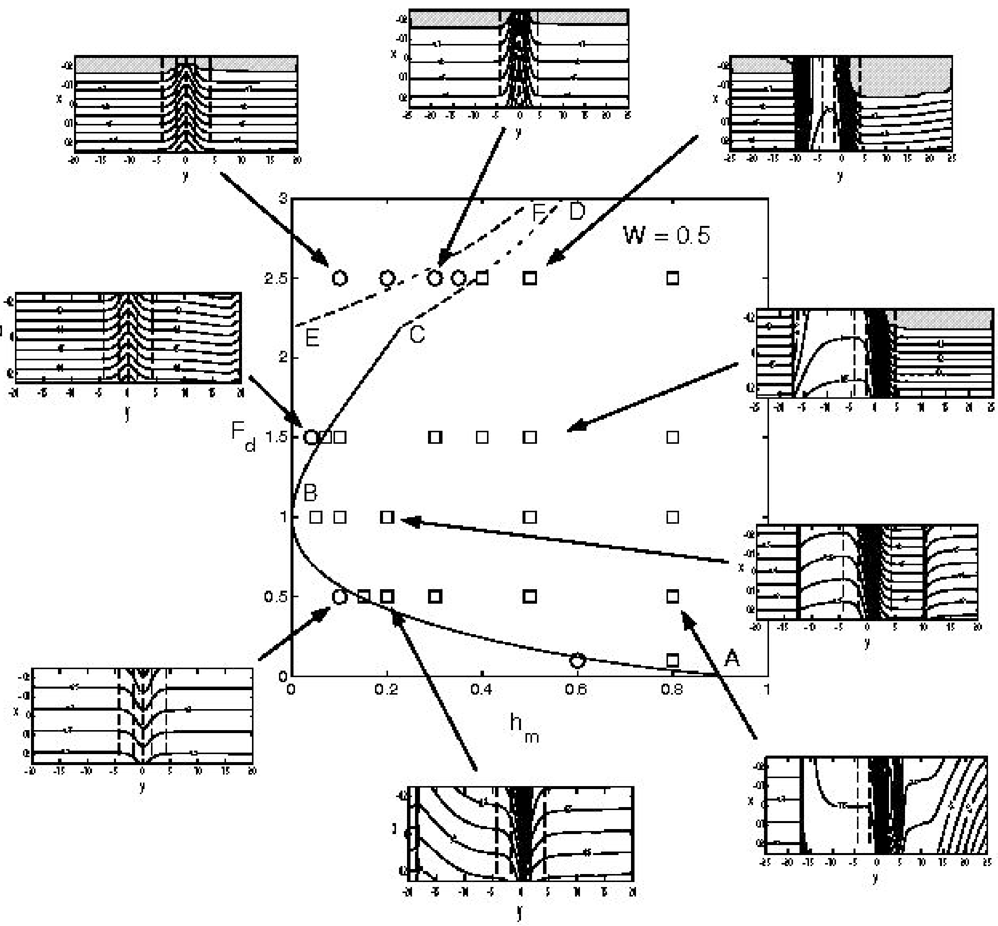


Figure 2. Regime diagram for upstream influence for a channel of width w = 0.5. See text for explanation.

Using the semigeostrophic approximation and the assumption of uniform potential vorticity we have developed a theory for the critical obstacle height above which upstream influence and hydraulic control at the topographic crest is achieved for rotating flow in a uniform channel. This height is a function of the initial flow Froude number (defined using the appropriate characteristic speeds in the rotating system) and the channel width *w* relative to the deformation radius based on the upstream potential depth. The theory is supplemented by numerical solutions of the full shallow water equations to explore the temporal development of the flow.

Figure 2 shows an example of the regime diagram for a channel of width $w = 0.5$. The curve *ABCD* is the critical curve for upstream influence from the semigeostrophic theory. Values of (F_d, h_m) to the right of this curve lead to upstream influence and hydraulically controlled flows. The solid part of the curve (*ABC*) indicates that the initial geostrophic flow is attached to both channel walls and the dashed segment (*CD*) indicates flow separated from the left wall. Points lying between the dashed segments *EF* and *CD* experience no upstream influence, but will attach briefly over the sill. Also shown in the figure by the symbols are results from the numerical solutions. The circles indicate no permanent upstream influence observed and the squares indicate upstream influence. The inset figures are contour plots of the free surface height at later stages of evolution from numerical runs chosen to illustrate some of the features observed. The gray shaded regions indicate patches of "dry" channel (zero layer depth) and the dashed lines running across the channel (x direction in these figures) indicate location of the topography, centered at $y = 0$. In all cases the initial flow is in the positive y direction. The insets show example of upstream propagating bores, downstream jumps (in flow depth and width), downstream separated supercritical flows and downstream recirculation.

We have found the semigeostrophic theory to give a good prediction of the occurrence of upstream influence and establishment of hydraulically controlled flows for narrow channels. As the channel widens the quantitative agreement decreases, but the qualitative behavior is recovered. Other flow features such as upstream propagating rarefying intrusions and recirculation over the sill crest emerge.

IMPACT / APPLICATIONS

The work on the hydraulics of rotating flows adds to the understanding of the role of straits and sills in regulating flow within and between abyssal basins, marginal seas, etc. The work on the dynamics of shocks and bores in rotating flows is applicable not only the issue of time-dependence in rotating hydraulic flows, but also to nonlinear wave dynamics in straits and coastal zones, including the dynamics of the marine boundary layer along the California coast. Thought the focus to date has been on single layer flows, the results gained so far should help in the interpretation of stratified flows. It is also worth pointing out that the fully nonlinear shocks and bores are the hydrostatic limit of the (typically) weakly nonlinear solitary wave and undular bore solutions. As such they may provide information on the initial, or boundary, conditions which result in solitary waves in the far field.

TRANSITIONS

None.

RELATED PROJECTS

None

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